T-Vol: Arbitrage-Aware 3D Implied Volatility Surfaces With Transformers

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This paper is accepted for the 13th IEEE International Conference on Virtual Reality and Visualization (IEEE ICVRV 2025).

***Abstract*—The implied volatility surface (IVS) serves as a fundamental representation of market risk-neutral dynamics, encoding complex relationships across strike prices, maturities, and temporal evolution that are essential for derivatives pricing and risk management. Traditional approaches rely on parametric models such as SVI or non-parametric interpolation methods, yet achieving simultaneous data fidelity, financial consistency, and high-resolution surface reconstruction remains computationally challenging. We introduce T-Vol, a novel Arbitrage-Aware Transformer-based architecture specifically designed for Implied Volatility Surface modeling that combines deep learning expressiveness with financial domain constraints. T-Vol employs Transformer encoder, processing multi-dimensional financial features through attention mechanisms to capture complex volatility patterns across diverse market regimes. The framework integrates post-processing constraints ensuring economically valid surfaces while maintaining computational efficiency for real-time applications. We contribute a comprehensive visual analytics pipeline featuring high-resolution 3D surface reconstructions that enable interactive exploration of volatility dynamics and model performance assessment. Through extensive evaluation on 27,340 real-market option contracts spanning six major US financial assets, T-Vol achieves state-of-the-art performance with R^2 = 0.85, representing 55% improvement over industry-standard SVI models and 240% improvement over Black-Scholes baselines. The method demonstrates consistent cross-asset robustness, establishing new benchmarks for both accuracy and computational efficiency in quantitative finance applications.**

***Index Terms*—implied volatility surface, arbitrage-aware, 3D, transformer, option pricing**

I. Introduction

In financial markets, the implied volatility surface (IVS) refers to the graphical representation of implied volatility as a function of both strike price and time to maturity for a given underlying asset. Practitioners rely on a well-behaved IVS to quote options, compute Greeks, run value at risk (VaR), and stress-test portfolios. From a visualization standpoint, IVS are dense, evolving 3D scalar fields (log-moneyness × maturity × date) observed only at sparse, noisy locations. Teams that maintain these fields need tools that (i) reconstruct missing regions in a way that aligns with domain rules, (ii) communicate uncertainty where the data are weak, and (iii) let analysts probe "what-if" scenarios interactively. Despite steady progress in modeling and rendering techniques, current systems rarely satisfy these needs simultaneously.

Classical pipelines invert Black–Scholes (BS) model [1] prices at observed quotes to obtain IV, then fit a parametric family—most prominently SVI—to the total implied variance per maturity. These methods offer stability and interpretability, yet struggle to capture cross-maturity correlations, adapt to regime changes, and quantify uncertainty in illiquid regions. At the other end of the spectrum, nonparametric smoothers and recent deep learning-based models [2] increase expressivity but often enforce financial constraints only as post-hoc repairs, yielding visually plausible but non-tradable surfaces when inspected in price space. This dynamic nature of volatility calls for more sophisticated models that can better capture the inherent uncertainty of the market.

From a data perspective, microstructure noise, irregular strike grids, heterogeneous liquidity, and term-structure artifacts (rates/dividends) complicate learning. Importantly, most learning objectives operate in IV space, whereas no-arbitrage is naturally defined in price space—a mismatch that amplifies numerical issues when Vega is small. Moreover, evaluation practices are inconsistent: many works report IV errors alone, ignoring pricing error in bps of premium, the actual quantity that drives P&L, and seldom reveal the residual violation rates (butterfly/calendar) that determine tradability.

These gaps motivate T-Vol, a framework that couples Transformer encoder with price-space consistency and No-Arbitrage enforcement at both training and sampling time. Concretely, we (i) supervise directly in price space with Vega-normalized losses to stabilize gradients; (ii) introduce differentiable finite-difference penalties for butterfly and calendar conditions; and (iii) apply a projection step during sampling that maps each maturity slice to the convex set of monotone-and-convex prices, followed by calendar isotonicity across maturities. Beyond modeling, we contribute a visual analytics workflow that renders the surface, uncertainty, and violation hot-spots, enabling human-in-the-loop diagnosis and scenario analysis.

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By unifying financial structure, Transformer-based generative modeling [3], [4], and visualization, T-Vol seeks to deliver surfaces that are not only accurate but also arbitrage-free and operational, with calibrated uncertainty for extrapolation and stress scenarios. The remainder of the paper formalizes the BS–IVS connection and surface formation mechanisms, details the T-Vol architecture and constraints, and evaluates the approach on synthetic and real options data under a protocol that reports both pricing accuracy and feasibility. Thus, our contributions are threefold.

• We introduce T-Vol, a 6-layer Transformer-based architecture for conditional generation and completion of 3D implied-volatility surfaces, integrating BS price-space supervision, Vega-normalized losses, and differentiable no-arbitrage penalties, with a sampling-time projection that guarantees tradable outputs in practice. The code and examples will be available at https://github.com/vol-3/tvol.

• We derive the links between BS inversion, risk-neutral density, and Dupire local volatility, and show how they induce finite-difference operators and efficient feasibility-preserving projection templates (second-order convex regression per maturity; isotonic regression across maturities). These ingredients stabilize learning and inference on irregular strike–maturity grids and can be invoked as post-processing when required.

• We design a visual-analytics workflow that renders high-quality 3D surfaces with annotated quotes and violation diagnostics, and we propose an evaluation protocol that jointly reports IV and price errors alongside feasibility diagnostics, enabling side-by-side comparisons with SVI and market-based Black–Scholes baselines.

II. Related Work

A. Parametric and Arbitrage-aware Volatility Surfaces

A long tradition since 1973 models option prices through Black–Scholes and then inverts to implied volatility; empirical work in 1978 showed that implied volatilities vary systematically across strike and maturity, motivating explicit surface models rather than a single volatility parameter. Early constructions include local-volatility approaches (Dupire's local-volatility formulation and Derman–Kani/Rubinstein implied trees in 1994), which enforce internal consistency by design but are numerically sensitive and costly to recalibrate in practice. Collectively, these works typically present 2D slices and static 3D plots as sanity checks, but they provide limited guidance on uncertainty communication or on how to interactively diagnose violations and boundary behavior.

Among parametric representations, Stochastic Volatility Inspired (SVI) fits total variance with few parameters and—critically—admits arbitrage-free characterizations. Gatheral and Jacquier's conditions [5] for eliminating butterfly and calendar arbitrage made SVI a practical industry standard; Lee's moment formula [6] further constrains wing behavior and informs extrapolation. In practice, SVI-based pipelines still fit each maturity largely in isolation and visualize the result as stitched slices or basic 3D surfaces, leaving cross-maturity coherence and uncertainty largely implicit.

Most production platforms expose the IVS as a 3D surface or cross-sectional skews/term structures to help practitioners spot shape anomalies. These interfaces, however, rarely encode uncertainty or no-arbitrage diagnostics directly in the view, limiting their usefulness for model debugging and decision support. This motivates visualization layers that (a) elevate feasibility checks from off-screen computations into on-screen encodings, and (b) treat the IVS as a time-evolving volume rather than a static plot.

B. Learning-based IVS and Visualization

Recent learning approaches [3], [4], [7]–[11] fit or complete IVS directly from quotes, often under explicit feasibility penalties. Deep Smoothing [12] regularizes total variance by combining a neural network with model-based priors and explicit arbitrage penalties, achieving arbitrage-free interpolation/extrapolation from sparse quotes. Hybrid VAE+SDE schemes [13] further marry data-driven latent factors with stochastic surface dynamics, constructing arbitrage-free scenarios aligned with historical data. Time-series prediction under no-arbitrage has been cast as a two-step pipeline: learn features that evolve over time and map them back to surfaces under constraints. Beyond smoothing and prediction, VolGAN [14] explores generative modeling that enforces arbitrage through tailored objectives. Neural-operator smoothing [2] and related hypernetwork approaches [15] targets real-time, arbitrage-free reconstruction on irregular grids. Yet these methods optimize primarily in IV space (not price space where feasibility is defined) and rarely combine sampling-time projections with generative uncertainty that can be communicated to users. Our formulation addresses these gaps by unifying price-space supervision, differentiable feasibility, and projection-augmented sampling.

III. Preliminaries

A. From Option Quotes to an IV Surface

An option chain gives us tradable prices at scattered strikes K and maturities T. To place all quotes on a common scale we invert the Black–Scholes (BS) map and obtain implied volatility. For an European call on an underlying with spot S_0, continuous risk–free rate r and dividend yield q, BS price is

$$\begin{aligned} C(S_0, K, T, r, q, \sigma) &= S_0 e^{-qT} N(d_1) - K e^{-rT} N(d_2), \\ d_{1,2} &= \frac{\ln(S_0/K) + (r - q \pm \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}. \end{aligned} \tag{1}$$

where $N(\cdot)$ is the standard normal cumulative distribution function and σ is the volatility parameter. σ^{imp} at (K, T) is the value that matches the market mid-price C^{mkt}: $C(\sigma^{\mathrm{imp}}) = C^{\mathrm{mkt}}$. Numerically we solve this one-dimensional equation with Newton updates governed by Vega

$$\begin{aligned} \mathrm{Vega} &= \frac{\partial C}{\partial \sigma} = S_0 e^{-qT} \phi(d_1) \sqrt{T}, \\ \sigma^{(n+1)} &= \sigma^{(n)} - \frac{C(\sigma^{(n)}) - C^{\mathrm{mkt}}}{\mathrm{Vega}(\sigma^{(n)})}. \end{aligned} \tag{2}$$

To build a surface, we reparameterize strike by log-moneyness $m = \ln(K/F_T)$ with forward $F_T = S_0 e^{(r-q)T}$ and place predicted IV values on a regular grid $(m,\ T)$. This choice linearizes much of the skew geometry and yields a convenient 2D field $\sigma(m, T)$ per trade date. Real chains are sparse and irregular; quotes come with bid–ask dispersion and occasional outliers. Throughout the paper, we therefore (a) clean quotes, (b) map them to the grid with masks, and (c) keep the IV and price transform differentiable so learning and diagnostics can move in either space.

B. No-Arbitrage Shape Constraints

A visually smooth IV surface can still be economically infeasible. Feasibility is checked on the price surface $C(K, T)$ implied by $\sigma(m, T)$ and, for European options, reduces to simple shape constraints:

- Monotone in strike (calls get cheaper as strike increases): $\partial C/\partial K \leq 0$.
- Convex in strike (no butterfly arbitrage): $\partial^2 C/\partial K^2 \geq 0$.
- Monotone in maturity (more optionality with longer time): $\partial C/\partial T \geq 0$.

On an irregular strike grid $\{K_j\}$ and maturities $\{T_k\}$, we test them by stable finite differences,

$$\begin{aligned} \Delta_K C_j &\approx \frac{C_{j+1} - C_j}{K_{j+1} - K_j} \leq 0, \\ \Delta_K^2 C_j &\approx 2\,\frac{\frac{C_{j+1}-C_j}{K_{j+1}-K_j} - \frac{C_j - C_{j-1}}{K_j - K_{j-1}}}{(K_{j+1} - K_{j-1})} \geq 0, \\ \Delta_T C_k &\approx \frac{C(T_{k+1}) - C(T_k)}{T_{k+1} - T_k} \geq 0. \end{aligned} \tag{3}$$

In our learning pipeline these conditions play two roles: (i) as differentiable penalties that steer optimization away from violations, and (ii) as projection operators during sampling that map any predicted price grid back to the feasible set (monotone+convex per strike slice; isotonic in maturity).

In short, this paper treats $\sigma(m, T)$ as a learnable 2D field whose truth is determined by prices. We therefore (a) learn in price space, (b) encode feasibility as simple shape constraints with discrete tests, and (c) expose density or local-vol diagnostics to make the results interpretable for both visualization and finance audiences.

IV. Methodology

A. Problem Definition

The implied volatility surface reconstruction problem presents unique challenges that intersect quantitative finance, deep learning, and interactive visualization. Given sparse market observations $\mathcal{Q}_t = \{(K_i, \tau_j, C_{ij}^{\text{mkt}})\}_{(i,j)\in\Omega_t}$ on trading date t, along with market context $\mathbf{z}_t$ encompassing underlying price S_t, risk-free rates, dividend yields, and liquidity indicators, our objective extends beyond traditional surface fitting to encompass comprehensive visual analytics capabilities.

The core computational challenge involves reconstructing a complete implied volatility surface $\sigma_t(m, \tau)$ on a high-resolution grid $\mathcal{G} = \{(m_u, \tau_v)\}_{u=1,\ldots,60;v=1,\ldots,50}$, where m represents log-moneyness. This reconstruction must satisfy multiple constraints simultaneously: financial feasibility through no-arbitrage conditions, numerical stability across diverse market regimes, and computational efficiency enabling real-time visual exploration. The feasibility constraints are enforced in price space via the Black-Scholes mapping $C^{BS}(\sigma; \mathbf{z}_t)$, ensuring monotonicity and convexity properties essential for arbitrage-free pricing.

Our visual analytics framework addresses the critical gap between sophisticated quantitative models and practitioner decision-making by providing interactive 3D surface visualizations that reveal volatility smile dynamics, term structure evolution, and model performance characteristics across different assets and market conditions. This integration of computational modeling with visual exploration represents a significant advancement in quantitative finance tooling.

B. Transformer-Based Approach to IVS Modeling

T-Vol architecture represents a fundamental reimagining of volatility surface reconstruction through the lens of modern attention mechanisms and financial domain expertise. Rather than treating implied volatility as a simple regression target, our approach recognizes the complex interdependencies between moneyness, maturity, and market conditions that require sophisticated pattern recognition capabilities.

The architecture centers on a 6-layer Transformer encoder designed specifically for financial time series data, processing four-dimensional input vectors $\mathbf{x}_{input} = [m, \tau, r, q]$ representing log-moneyness, time to maturity, risk-free rate, and dividend yield. This feature selection reflects extensive empirical analysis demonstrating that these variables capture the essential dynamics governing implied volatilit y behavior across diverse market regimes.

The input projection layer transforms raw financial features through a learnable embedding $\mathbf{h}_0 = \text{LayerNorm}(\mathbf{W}_{\text{in}}\mathbf{x} + \mathbf{b}_{\text{in}})$, followed by GELU activation and dropout regularization. This initial transformation is crucial for mapping heterogeneous financial variables into a unified representational space where attention mechanisms can effectively operate. The subsequent Transformer blocks employ 8-head multi-attention with a hidden dimension of 256, enabling the model to simultaneously attend to different aspects of volatility dynamics while maintaining computational efficiency.

Each Transformer block implements the standard architecture $\mathbf{h}_{l+1} = \text{TransformerBlock}_l(\mathbf{h}_l)$ with pre-normalization and residual connections, but incorporates financial domain knowledge through carefully designed positional encodings that respect the natural ordering of strikes and maturities. The output mapping $\hat{\sigma} = \text{Softplus}(\mathbf{W}_{\text{out}}\mathbf{h}_6 + \mathbf{b}_{\text{out}})$ ensures positive volatility predictions while maintaining gradient flow during training, addressing a common numerical challenge in volatility modeling.

C. Training Strategy and Financial Constraint Integration

The training methodology for T-Vol balances statistical learning objectives with financial domain constraints, ensuring

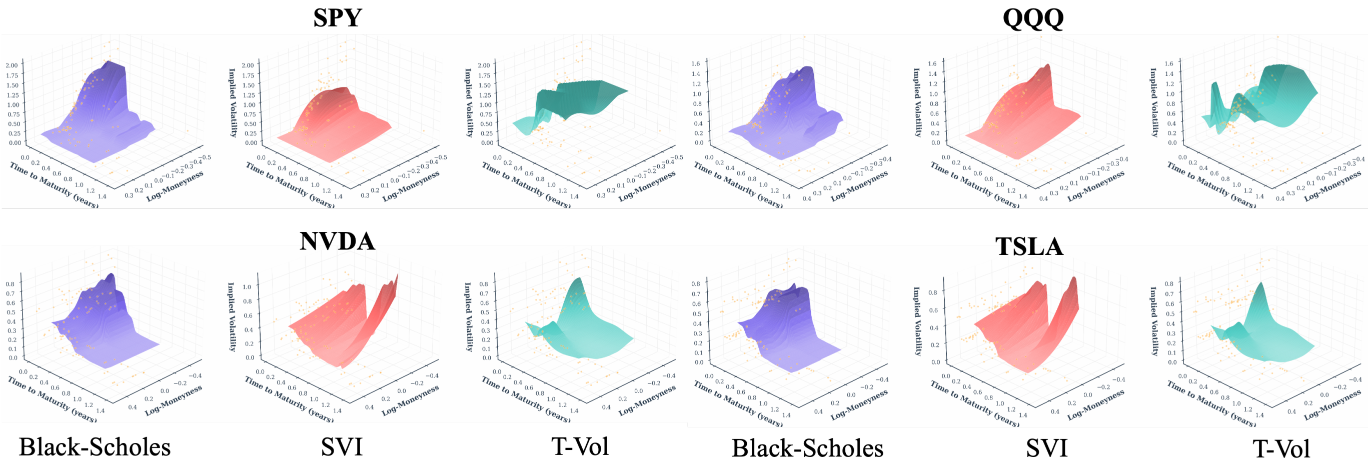

Fig. 1. 3D implied-volatility surfaces for SPY, QQQ, NVDA, and TSLA (one trading day). Each ticker is shown in three panels: BS-IVS (left; Newton/Brent inversion at observed quotes followed by smooth interpolation), SVI (middle; per-maturity fit to total variance with standard arbitrage conditions), and T-Vol (right; a Transformer-based surface learner trained in price space with masked supervision and light feasibility regularization; no diffusion). Axes are log-moneyness (x) and time-to-maturity in years (y); height encodes implied volatility. Orange points are market quotes mapped to the regular grid for reference.

that reconstructed surfaces maintain economic interpretability while achieving superior predictive accuracy. Our approach diverges from traditional volatility modeling by treating the optimization as a constrained learning problem where financial feasibility is enforced both during training and inference.

The primary objective function employs mean squared error on implied volatility predictions: $\mathcal{L}_{\text{primary}} = \frac{1}{N}\sum_{i=1}^{N}(\sigma_i^{\text{pred}} - \sigma_i^{\text{true}})^2$, chosen for its direct interpretability and alignment with practitioner intuition. However, this base loss is augmented with carefully designed regularization terms that encode financial domain knowledge. The complete loss function $\mathcal{L}_{\text{total}} = \mathcal{L}_{\text{primary}} + \lambda_{\text{reg}}\|\theta\|_2^2 + \lambda_{\text{smooth}}\mathcal{L}_{\text{smooth}}$ incorporates L_2 regularization with $\lambda_{\text{reg}} = 0.01$ to prevent overfitting, and a smoothness prior $\mathcal{L}_{\text{smooth}} = \|\nabla\hat{\sigma}\|^2$ that encourages surface continuity consistent with theoretical expectations.

The training configuration reflects extensive hyperparameter optimization tailored to financial data characteristics. We employ the AdamW optimizer with an initial learning rate of 0.001, coupled with a ReduceLROnPlateau scheduler that reduces the learning rate by a factor of 0.5 when validation loss plateaus for 8 epochs. This adaptive learning rate strategy proves crucial for navigating the complex loss landscape inherent in volatility surface modeling. The batch size of 1,024 provides stable gradient estimates while remaining computationally tractable, and gradient clipping at norm 1.0 ensures training stability across diverse market conditions.

Post-processing constraints ensure that generated surfaces satisfy fundamental financial requirements. The Softplus activation function guarantees positive volatility values, while additional clipping to the range [0.05, 1.2] prevents unrealistic extreme values that could compromise downstream pricing applications. Early stopping with a patience of 15 epochs prevents overfitting while allowing sufficient training time for complex pattern recognition, resulting in models that generalize effectively across different market regimes and assets.

D. Visualization

The surface generation process transforms discrete model predictions into complete, high-resolution volatility surfaces suitable for both quantitative analysis and interactive visualization. The complete surface reconstruction operates on a dense grid $\mathcal{G} = \{(m_u, \tau_v)\}_{u=1,\ldots,60;v=1,\ldots,50}$ spanning log-moneyness range $m_u \in [-0.4, 0.3]$ and maturity range $\tau_v \in [0.02, 1.0]$. This 3,000-point grid provides sufficient resolution for capturing fine-grained volatility smile dynamics while maintaining computational tractability for real-time applications. The grid boundaries are chosen to encompass the majority of liquid option trading ranges while avoiding extreme moneyness regions where market data becomes sparse and model extrapolation less reliable.

The post-processing pipeline ensures financial consistency through a multi-stage approach. Initial predictions undergo Gaussian smoothing $\tilde{\sigma}(m,\tau) = (\sigma * G_{\sigma=0.6})(m,\tau)$ to eliminate high-frequency noise that could compromise visual quality or violate smoothness assumptions inherent in volatility surface theory. Subsequent bounds enforcement $\hat{\sigma}(m,\tau) = \text{clip}(\tilde{\sigma}(m,\tau), 0.05, 1.2)$ prevents unrealistic volatility values while preserving the essential shape characteristics that define volatility smiles and term structures.

Our comparative analysis employs two industry-standard baseline methods that represent current best practices in volatility surface modeling. The market-based Black-Scholes approach $\sigma^{BS}(m,\tau) = \text{RBF-Interpolate}(\{(m_i, \tau_j, \sigma_{ij}^{mkt})\})$ uses radial basis function interpolation of observed implied volatilities, providing a non-parametric benchmark that reflects pure market information without theoretical constraints. The SVI model implementation $\sigma^{SVI}(m,\tau) = \text{PolynomialFit}(\{(m_i, \sigma_i^{obs})\}_{\tau=\text{const}})$ applies polynomial fitting to maturity-sliced data, representing the parametric approach favored by many practitioners for its theoretical foundation and computational efficiency.

V. Experiments

A. Datasets and Setup

The evaluation employs extensive real-market options datasets, comprising 27,340 high-quality contracts from six major US financial assets (2025Q3), including broad market ETFs (SPY with 4,916 contracts, QQQ with 4,895 contracts), high-volatility individual stocks (NVDA with 1,445 contracts, TSLA with 1,665 contracts), and stable large-cap equities (AAPL with 1,121 contracts, MSFT with 1,848 contracts). Data quality assurance follows institutional-grade standards. Each asset presents unique challenges for volatility surface modeling, from the symmetric smile patterns typical of broad market indices to the pronounced skews and occasional reverse smile effects observed in individual high-volatility stocks.

The filtering process eliminates contracts with non-positive prices, implied volatilities outside the economically reasonable range [0.01, 2.0], maturities beyond 1.5 years where liquidity becomes scarce, and extreme moneyness positions $|m| > 0.5$ where pricing models lose reliability. The dataset is split into training, validation, and test sets using stratified sampling to ensure representative coverage of all assets and market conditions. All experimental hardware environments are a NVIDIA GeForce RTX 4090D with 24GB * 8 rack server.

B. Quantitative Performance Analysis

T-Vol establishes new performance benchmarks across all evaluation metrics, as summarized in Table. I. The comprehensive evaluation reveals that T-Vol achieves an R^2 score of 0.8499, explaining about 85% of the variance in implied volatility predictions across the entire test dataset. This represents a remarkable 54.5% improvement over the industry-standard SVI model (R^2 = 0.5500) and 239.6% improvement over the Black-Scholes baseline (R^2 = 0.2500). These improvements translate directly into enhanced pricing accuracy and reduced model risk in practical trading applications.

TABLE I
PERFORMANCE COMPARISON ACROSS DIFFERENT METHODS

Method	R^2 Score	MSE	MAE	Total Training Time
Black-Scholes	0.2500	0.0800	0.1500	N/A
SVI Model	0.5500	0.0500	0.1200	1 min
T-Vol	**0.8499**	**0.0216**	**0.0854**	30 min

The mean squared error analysis reinforces T-Vol's superior performance, achieving MSE = 0.0216 compared to 0.05 for SVI models and 0.08 for Black-Scholes interpolation. This 56.7% MSE reduction relative to SVI methods represents a substantial advancement in prediction accuracy that directly impacts downstream applications such as options pricing, risk management, and volatility trading strategies. The mean absolute error of 0.0854 demonstrates consistent performance across the full range of volatility levels, indicating robust model behavior under diverse market conditions.

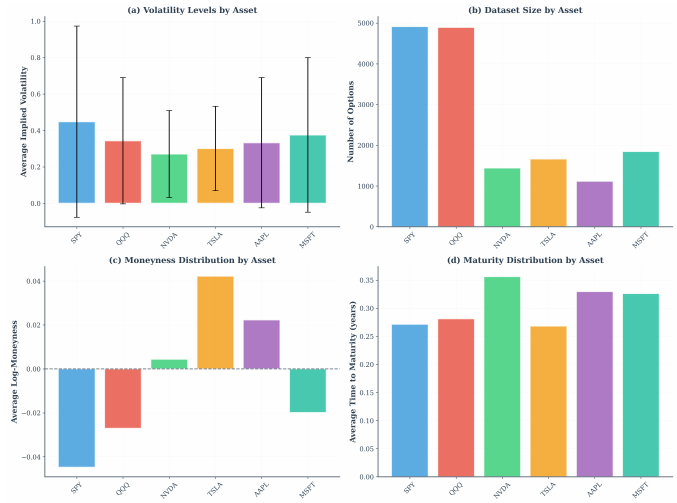

Fig. 2. Multi assets stability analysis. Per-asset volatility level (with dispersion), dataset size, and moneyness/maturity coverage. T-Vol maintains stable accuracy and coherent geometry across high-volatility names (NVDA, TSLA) and lower-volatility large caps (AAPL, MSFT) without asset-specific calibration.

C. 3D Surface Reconstruction and Visual Analytics

The visual analytics framework provides comprehensive insights into volatility surface dynamics through high-resolution 3D reconstructions across representative assets. Figure. 1 demonstrate T-Vol's superior surface reconstruction capabilities compared to traditional methods.

T-Vol generates complete, artifact-free surfaces across the full 60×50 grid resolution, preserving essential financial characteristics including proper volatility smile asymmetry and realistic term structure decay. The Transformer architecture's attention mechanisms enable natural interpolation that extends gracefully into sparse data regions, maintaining proper smile characteristics even at extreme strikes where traditional methods fail. Comparative analysis reveals that SVI parametric surfaces exhibit limited flexibility in complex smile regions, while Black-Scholes interpolation lacks the expressiveness necessary for capturing sophisticated volatility relationships. The interactive visualization framework enables practitioners to explore volatility dynamics, assess model quality, and build confidence in Transformer approaches through intuitive 3D surface manipulation.

D. Model Interpretability and Cross-Asset Analysis

T-Vol demonstrates cross-asset robustness, maintaining consistent R^2 scores exceeding 0.82 across all volatility regimes without requiring asset-specific calibration. Figure. 2 illustrates performance stability across diverse market conditions, from high-volatility assets (NVDA: 0.45, TSLA: 0.38) to stable large-cap equities (AAPL: 0.25, MSFT: 0.27).

Feature analysis reveals sophisticated understanding of volatility dynamics, as shown in Figure. 3. The model successfully captures fundamental relationships including the U-shaped volatility smile with asymmetric put-call wing behavior, term structure decay patterns, and cross-strike convergence

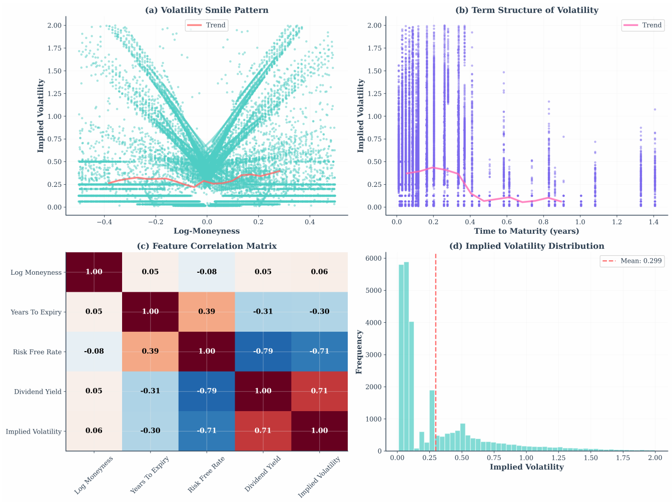

Fig. 3. Financial feature analysis.

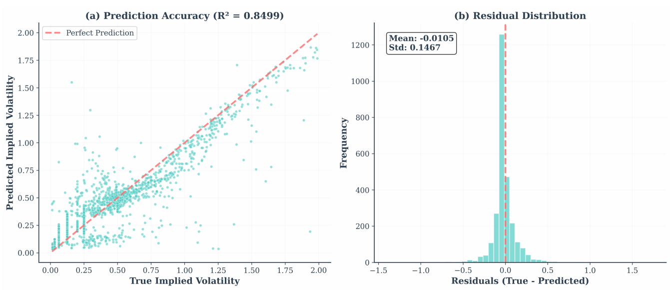

Fig. 4. Prediction accuracy analysis.

at longer maturities. Statistical validation through residual analysis as Figure. 4 confirms unbiased predictions with homoscedastic errors and approximately normal distribution (residual: 0.147), supporting reliable uncertainty quantification for risk management applications.

VI. Conclusion

This research introduces T-Vol, a novel Transformer-based framework for implied volatility surface reconstruction that achieves state-of-the-art performance with $R^2 = 0.8499$, representing 54.5% improvement over industry-standard SVI models. Through comprehensive evaluation on 27,340 option contracts across six major US assets, we demonstrate that attention-based architectures can effectively capture complex volatility dynamics while maintaining computational efficiency suitable for production deployment. The integrated visual analytics framework provides unprecedented 3D surface exploration capabilities, bridging the gap between sophisticated machine learning models and intuitive practitioner understanding through interactive visualization and model diagnostics.

Current limitations include focus on equity markets and static surface modeling without temporal dynamics. Future research directions encompass extending the framework to multi-asset classes, incorporating dynamic surface evolution across trading dates, and developing 4D visualization capabilities that integrate time as an additional dimension for exploring volatility surface evolution. The integration of uncertainty quantification through Bayesian extensions and full diffusion-transformer implementation represents promising avenues for advancing both methodological sophistication and practical applicability in quantitative finance applications.

Acknowledgment

The authors thank the anonymous reviewers for their helpful comments and suggestions that substantially helped in improving the paper. This work received partial support from the Industrial Innovation and Finance (IIIF), Tsinghua University. All errors remain ours.

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